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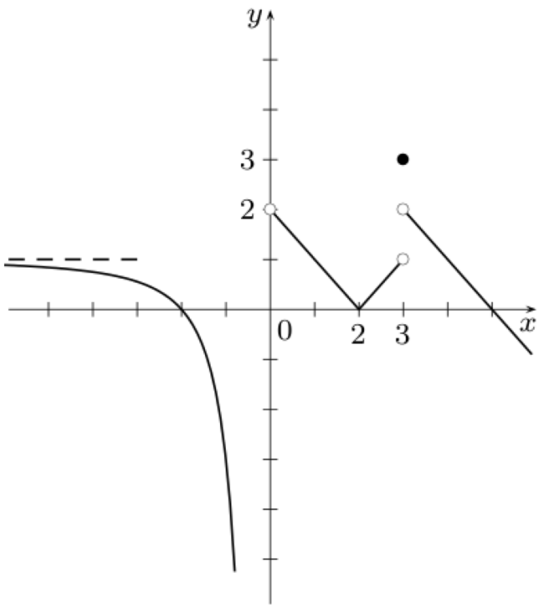
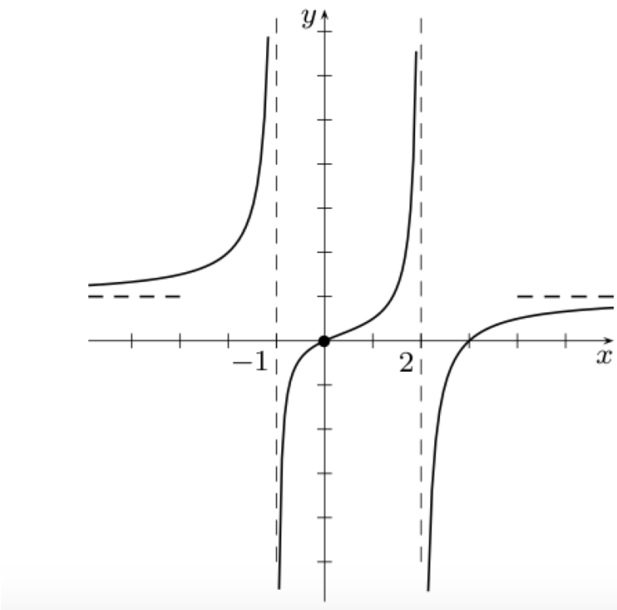
MAM1020S **Tutorial 5** **August 2017**

1. Use the graph of the function f to answer each question. Some times, there is the answer might not exist just specify when this is the case.



(a) $f(0)$ =does not exist.
(b) $f(2) = 0$
(c) $f(3) = 3$
(d) $lim_{x\to 0^-} f(x) = -\infty$
(e) $lim_{x\to 0^+} f(x) = 2$
(f) $lim_{x\to 0} f(x)$ =does not exists
(g) $lim_{x\to 3^+} f(x) = 2$
(h) $lim_{x\to 3^-} f(x) = 1$
(i) $lim_{x\to 3} f(x)$ =does not exist
(j) $lim_{x\to -\infty} f(x) = 1$

2. Use the graph of the function f to answer each question. Some times, there is the answer might not exist just specify when this is the case.



(a) $f(0) = 0$
(b) $f(2)$ =does not exist
(c) $f(3) = 0$
(d) $lim_{x\to -1^+} f(x) = -\infty$
(e) $lim_{x\to -1^-} f(x) = \infty$
(f) $lim_{x\to -1} f(x)$ =does not exist
(g) $lim_{x\to 0} f(x) = f(0) = 0$
(h) $lim_{x\to 2^+} f(x) = -\infty$
(i) $lim_{x\to 2^-} f(x) = \infty$
(j) $lim_{x\to 2} f(x)$ =does not exist
(k) $lim_{x\to -\infty} f(x) = 1$
(l) $lim_{x\to \infty} f(x) = 1$

3. Evaluate each limit using algebraic techniques. Some times, there is the answer might not exist just specify when this is the case. **SEE HELP NOTE FOR HOW YOU SHOULD DO TO THINK AND ANSWER THIS QUESTIONS**

1. $lim_{x\to 0}\frac{x^2-25}{x^2-4x-5}$
Solution:
How you should think in your draft/head: Well if I plug 0 in the expression there is no problem the function is defined and continuous and well defined. Cool I have not so much work to do then.
How you can answer:
We see that 0 is part of the domain of definition of the continuous rational function defined by $\frac{x^2-25}{x^2-4x-5}$ thus
$$lim_{x\to 0}\frac{x^2-25}{x^2-4x-5} = \frac{0-25}{0-4\times 0-5} = 5$$

2. $lim_{x\to 5}\frac{x^2-25}{x^2-4x-5}$
How you should think in your draft/head: Well if I plug 5 in the expression the denominator and numerator are 0. I have a indeterminate, if I see the help notes for limits, this should simplify well so I can get a determinate form. To get an determinate form first work of your function alone then take the limit you will avoid mistake or incoherence.
How you can answer:
Note that
$$\frac{x^2-25}{x^2-4x-5} = \frac{(x-5)(x+5)}{(x+1)(x-5)} = \frac{x+5}{x+1}$$
The right hand of the equality is now defined and continuous at 5, thus
$$lim_{x\to 5}\frac{x^2-25}{x^2-4x-5} = lim_{x\to 5}\frac{x+5}{x+1} = \frac{5}{3}$$

3. $lim_{x\to 1}\frac{7x^2-4x-3}{3x^2-4x+1}$
How you should think in your draft/head: Well if I plug 1 in the expression the denominator and numerator are 0. I have a indeterminate, if I see the help notes for limits, this should simplify well so I can get a determinate form. To get an determinate form first work of your function alone then take the limit you will avoid mistake or incoherence.
How you can answer:
Note that
$$\frac{7x^2-4x-3}{3x^2-4x+1} = \frac{(x-1)(7x+3)}{(3x-1)(x-1)} = \frac{7x+3}{3x-1}$$
The right hand of the equality is now defined and continuous at 1, thus
$$lim_{x\to 1}\frac{7x^2-4x-3}{3x^2-4x+1} = lim_{x\to 1}\frac{7x+3}{3x-1} = 5$$

4. $lim_{x\to 2}\frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$
How you should think in your draft/head: Well if I plug 2 in the expression the denominator is zero and numerator are 0. THIS IS NOT AN INDETERMINATE FORM.
How you can answer:
Note that
$$lim_{x\to 2}x^4+5x^3+6x^2 = 2^4+5\times 2^3+6\times 2^2 > 0$$
and
$$x^2(x+1)-4(x+1) = (x+1)(x^2-4) = (x+1)(x-2)(x+2)$$
Clearly, this is positive when $x > 2$ and negative when $0 < x < 2$. Thus,
$$lim_{x\to 2^+}x^2(x+1)-4(x+1) = 0^+$$
Hence, using the tables
$$lim_{x\to 2^+}\frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)} = \infty$$
Also,
$$lim_{x\to 2^-}x^2(x+1)-4(x+1) = 0^-$$

Hence, using the tables

$$lim_{x\to 2^-}\frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}=-\infty$$

Thus the right and left limits do not coincide and $lim_{x\to 2}\frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$ does not exists.

5. $lim_{x\to -3}|x+1|+\frac{3}{x}$
How you should think in your draft/head: Well if I plug -3 there is not really nothing to worry about the function is defined are continuous at -3. So that is an easy question.
How you can answer:
Since the function defined by $|x+1|+\frac{3}{x}$ is continuous at -3 we have

$$lim_{x\to -3}|x+1|+\frac{3}{x}=|-3+1|+\frac{3}{-3}=2-1=1$$

6. $lim_{x\to 3}\frac{\sqrt{x+1}-2}{x^2-9}$
How you should think in your draft/head: Well if I plug 3 we get zero for denominator and numerator so I got indeterminate form. Since there is a square root maybe rationalizing help me to get rid of the square root.
How you can answer:

$$\frac{\sqrt{x+1}-2}{x^2-9}=\frac{\sqrt{x+1}-2}{x^2-9}\times\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}=\frac{x-3}{(x-3)(x+3)(\sqrt{x+1}+2)}=\frac{1}{(x+3)(\sqrt{x+1}+2)}$$

Since the function defined by $\frac{1}{(x+3)(\sqrt{x+1}+2)}$ is continuous at 3 we have

$$lim_{x\to 3}\frac{\sqrt{x+1}-2}{x^2-9}=lim_{x\to 3}\frac{1}{(x+3)(\sqrt{x+1}+2)}=\frac{1}{24}$$

7. $lim_{x\to 3}\frac{\sqrt{x^2+7}-3}{x+3}$
How you should think in your draft/head: Well if I plug 3 there is absolutely no problem the denominator is not 0. Great, it is a super easy question.
How you can answer: The function defined by $\frac{\sqrt{x^2+7}-3}{x+3}$ is well defined and continuous at 3 thus

$$lim_{x\to 3}\frac{\sqrt{x^2+7}-3}{x+3}=\frac{\sqrt{3^2+7}-3}{3+3}=\frac{8}{3}$$

8. $lim_{x\to 2}\frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}$
How you should think in your draft/head: Well if I plug 2 we get zero for denominator and numerator so I got indeterminate form. Since there is a square root maybe rationalizing help me to get rid of the square root.
How you can answer:

$$\begin{aligned}
\frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)} &= \frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}\times\frac{\sqrt{x^2+5}+(x+1)}{\sqrt{x^2+5}+(x+1)}\\
&= \frac{(x^2+2x-8)(\sqrt{x^2+5}+(x+1))}{x^2+5-(x+1)^2}\\
&= \frac{(x^2+2x-8)(\sqrt{x^2+5}+(x+1))}{4-2x}\\
&\quad (\textit{In your head: well the denominator and numerator are still } 0 \textit{ at } 2 \textit{ so I need to go on})\\
&= \frac{(x-2)(x+4)(\sqrt{x^2+5}+(x+1))}{-2(x-2)}\\
&= \frac{(x+4)(\sqrt{x^2+5}+(x+1))}{-2}
\end{aligned}$$

Since the function defined by $\frac{1}{(x+3)(\sqrt{x+1}+2)}$ is continuous at 3 we have

$$lim_{x\to 3}\frac{\sqrt{x+1}-2}{x^2-9}=lim_{x\to 3}\frac{1}{(x+3)(\sqrt{x+1}+2)}=\frac{1}{24}$$

9. $lim_{y\to 5}\left(\frac{2y^2+2y+4}{6y-3}\right)^{\frac{1}{3}}$

How you should think in your draft/head: Well if I plug 5, your function the function is well defined no problem this is also an easy question

How you can answer:

The function defined by $\left(\frac{2y^2+2y+4}{6y-3}\right)^{\frac{1}{3}}$ is well defined and continuous at 5 thus

$$lim_{y\to 5}\left(\frac{2y^2+2y+4}{6y-3}\right)^{\frac{1}{3}} = \left(\frac{2\times 5^2+2\times 5+4}{6\times 5-3}\right)^{\frac{1}{3}} = \frac{4}{3}$$

10. $lim_{x\to 0}\sqrt{2cos(x)-5}$

How you should think in your draft/head: Well no where near 0 this function is define we are taking the square root of a negative number even if I am very near 0.

How you can answer:

We know that for any $x\in\mathbb{R}$, $-1\leq cos(x)\leq 1$ thus $-7\leq 2cos(x)-5\leq -3$. So the function defined by $\sqrt{2cos(x)-5}$ is never defined over the real number and since we cannot speak about the function we cannot even speak about any limit.

11. $lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x}$

How you should think in your draft/head: Well when I plug 0 to the function we get 0 as numerator and denominator thus and undetermined function.

How you can answer:

We have

$$\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x} = \frac{-2x}{(3+x)(3-x)x} = \frac{-2}{(3+x)(3-x)}$$

The function defined by $\frac{-2}{(3+x)(3-x)}$ is well defined and continuous at 0. Thus

$$lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x} = lim_{x\to 0}\frac{-2}{(3+x)(3-x)} = \frac{-2}{9}$$

11. $lim_{x\to -6}\frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6}$

How you should think in your draft/head: Well when I plug 0 to the function we get 0 as numerator and denominator thus and undetermined function.

How you can answer:

We have

$$\begin{aligned}\frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6} &= \frac{x(2x+8)-(x^2-12)}{x(x^2-12)(x+6)}\\ &= \frac{x^2+8x+12}{x(x^2-12)(x+6)}\\ &= \frac{(x+2)(x+6)}{x(x^2-12)(x+6)}\\ &= \frac{x+2}{x(x^2-12)}\end{aligned}$$

The function defined by $\frac{x+2}{x(x^2-12)}$ is well defined and continuous at -6. Thus

$$lim_{x\to -6}\frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6} = lim_{x\to -6}\frac{x+2}{x(x^2-12)} = \frac{1}{36}$$

12. $lim_{x\to\infty}\sqrt{x^2-2}-\sqrt{x^2+1}$

How you should think in your draft/head: Quickly, you can intuitively think at infinity $\sqrt{x^2-2}$ should be ∞ and $-\sqrt{x^2+1}$ is $-\infty$. This is an indeterminate for sums. Since we have a square root MAYBE rationalizing could help lets try.

How you can answer:

We have

$$\begin{aligned}\sqrt{x^2-2}-\sqrt{x^2+1} &= \frac{(\sqrt{x^2-2}-\sqrt{x^2+1})(\sqrt{x^2-2}+\sqrt{x^2+1})}{(\sqrt{x^2-2}+\sqrt{x^2+1})}\\ &= \frac{-3}{(\sqrt{x^2-2}+\sqrt{x^2+1})}\end{aligned}$$

Since $lim_{x\to\infty}x^2-2 = lim_{x\to\infty}x^2+1 = \infty$ then using the composition rule since $lim_{X\to\infty}\sqrt{X} = \infty$, $lim_{x\to\infty}\sqrt{x^2-2} = lim_{x\to\infty}\sqrt{x^2+1} = \infty$. Thus,

$$lim_{x\to\infty}\sqrt{x^2-2}-\sqrt{x^2+1} = lim_{x\to\infty}\frac{-3}{(\sqrt{x^2-2}+\sqrt{x^2+1})} = 0$$

13. $lim_{x\to-\infty}\sqrt{x-2}-\sqrt{x}$ *How you should think in your draft/head:* Quickly, you can intuitively think at infinity $\sqrt{x-2}$ should be ∞ and $-\sqrt{x}$ is $-\infty$. This is an indeterminate for sums. Since we have a square root MAYBE rationalizing could help lets try.

    How you can answer:

    We have

    $$\begin{aligned}\sqrt{x-2}-\sqrt{x} &= \tfrac{(\sqrt{x-2}-\sqrt{x})(\sqrt{x-2}+\sqrt{x})}{(\sqrt{x-2}+\sqrt{x})} \\ &= \tfrac{-2}{(\sqrt{x-2}-\sqrt{x})}\end{aligned}$$

    Since $lim_{x\to\infty}x-2 = lim_{x\to\infty}x = \infty$ then using the composition rule since $lim_{X\to\infty}\sqrt{X} = \infty$, $lim_{x\to\infty}\sqrt{x-2} = lim_{x\to\infty}\sqrt{x^2+1} = \infty$.Thus,

    $$lim_{x\to\infty}\sqrt{x-2}-\sqrt{x} = lim_{x\to\infty}\frac{-2}{(\sqrt{x-2}-\sqrt{x})} = 0$$

14. $lim_{x\to 7}\sqrt{2x-14}$

    How you should think in your draft/head: There is a problem here because $\sqrt{2x-14}$ is not even defined close to 7 when $x < 7$.

    How you can answer: The function defined by $\sqrt{2x-14}$ is even not defined when x approaches 7 on the left. Thus this limit does not not exist.

15. $lim_{x\to 1^-}\sqrt[6]{3-3x}$

    How you should think in your draft/head: There is a problem here because $\sqrt[6]{3-3x}$ is not even defined close to 1 when $x < 1$.

    How you can answer: The function defined by $\sqrt[6]{3-3x}$ is even not defined when x approaches 1 on the left. Thus this limit does not not exist.

16. $lim_{x\to\infty}\frac{x^4-10}{4x^3+x}$ *How you should think in your draft/head:* This is easy it is a rational function indeterminate if I leave it like this. I know what to do.

    How you can answer:

    $$\frac{x^4-10}{4x^3+x} = x\frac{1-\frac{10}{x^4}}{4-\frac{1}{x^2}}$$

    Since $lim_{x\to\infty}\frac{10}{x^4} = lim_{x\to\infty}\frac{1}{x^2} = 0$, then

    $$lim_{x\to\infty}\frac{1-\frac{10}{x^4}}{4-\frac{1}{x^2}} = \frac{1-0}{4-0} = \frac{1}{4}$$

    and

    $$lim_{x\to\infty}x = \infty$$

    Thus,

    $$lim_{x\to\infty}\frac{x^4-10}{4x^3+x} = lim_{x\to\infty}x\frac{1-\frac{10}{x^4}}{4-\frac{1}{x^2}} = \infty$$

17. $lim_{x\to-\infty}\sqrt[3]{\frac{x-3}{5-x}}$

    How you should think in your draft/head: Here we have a composite with a rational function thus first we should find the limit of the rational function and then do the composite maybe.

    How you can answer:

    $$\frac{x-3}{5-x} = \frac{1-\frac{3}{x}}{-1+\frac{5}{x}}$$

    Since $lim_{x\to-\infty}\frac{3}{x} = lim_{x\to-\infty}\frac{5}{x} = 0$, then

    $$lim_{x\to-\infty}\frac{1-\frac{3}{x}}{-1+\frac{5}{x}} = \frac{1-0}{-1+0} = -1$$

and

$$lim_{x\to-\infty}x = -1$$

Moreover,

$$lim_{X\to-1}\sqrt[3]{X} = -1$$

since the function $\sqrt[3]{x}$ is continuous at -1. Thus, by the composite rule we get

$$lim_{x\to-\infty}\sqrt[3]{\frac{x-3}{5-x}} = lim_{x\to-\infty}\sqrt[3]{\frac{1-\frac{3}{x}}{-1+\frac{5}{x}}} = -1$$

18. $lim_{x\to\infty}\frac{3x^3+x^2-2}{x^2+x-2x^3+1}$
How you should think in your draft/head: rational function easy.
How you can answer:

$$\frac{3x^3+x^2-2}{x^2+x-2x^3+1} = \frac{x^3(3+1/x-2/x^3)}{x^3(1/x+1/x^2-2+1/x^3)} = \frac{(3+1/x-2/x^3)}{(1/x+1/x^2-2+1/x^3)}$$

$$lim_{x\to\infty}1/x = lim_{x\to\infty}-2/x^3 = lim_{x\to\infty}1/x^2 = lim_{x\to\infty}1/x^3 = 0$$

Thus

$$lim_{x\to\infty}\frac{3x^3+x^2-2}{x^2+x-2x^3+1} = lim_{x\to\infty}\frac{(3+1/x-2/x^3)}{(1/x+1/x^2-2+1/x^3)} = \frac{(3+0-0)}{(0+0-2+0)} = -\frac{3}{2}$$

19. $lim_{x\to\infty}\frac{x+5}{2x^2+1}$
How you should think in your draft/head: rational function easy.
How you can answer:

$$\frac{x+5}{2x^2+1} = \frac{x(1+5/x)}{x^2(2+1/x^2)} = \frac{1+5/x}{x(2+1/x^2)}$$

$$lim_{x\to\infty}2+1/x^2 = 2+0 = 2$$

and

$$lim_{x\to\infty}x = \infty$$

Thus

$$lim_{x\to\infty}x(2+1/x^2) = \infty$$

and since

$$lim_{x\to\infty}1+5/x = 1+0 = 1$$

we have using the limit tables,

$$lim_{x\to\infty}\frac{x+5}{2x^2+1} = lim_{x\to\infty}\frac{1+5/x}{x(2+1/x^2)} = 0$$

20. $lim_{x\to-\infty}cos(\frac{x^5+1}{x^6+x^5+100})$
How you should think in your draft/head: composite of cos and rational function that is easy.
How you can answer:

$$\frac{x^5+1}{x^6+x^5+100} = \frac{x^5(1+1/x^5)}{x^6(1+1/x+1/x^6)} = \frac{(1+1/x^5)}{x(1+1/x+1/x^6)}$$

$$lim_{x\to-\infty}1+1/x+1/x^6 = 1+0+0 = 1$$

and

$$lim_{x\to-\infty}x = -\infty$$

Thus

$$lim_{x\to-\infty}x(1+1/x+1/x^6) = -\infty$$

and since

$$lim_{x\to-\infty}1+1/x^5 = 1+0 = 1$$

we have using the limit tables,

$$lim_{x\to-\infty}\frac{x^5+1}{x^6+x^5+100} = lim_{x\to-\infty}\frac{(1+1/x^5)}{x(1+1/x+1/x^6)} = 0$$

Also, cos is continuous at 0 thus

$$lim_{X\to 0}cos(X) = cos(0) = 1$$

with $X = \frac{x^5+1}{x^6+x^5+100}$.
Using the composite rule we get that

$$lim_{x\to-\infty}cos(\frac{x^5+1}{x^6+x^5+100}) = 1$$

21. $lim_{x\to 2}\frac{2x}{x^2-4}$
How you should think in your draft/head: If I plug 2, I get $4 > 0$ on the numerator and 0 on the denominator. That is not an indeterminate, but I still need to separate left and right limits.
How you can answer:

$$lim_{x\to 2}2x = 4 > 0$$

and $x^2-4 = (x+2)(x-2)$ is positive when $x > 2$ and negative when $0 < x < 2$. Thus

$$lim_{x\to 2^+}x^2-4 = 0^+$$

and using the limit tables we get

$$lim_{x\to 2^+}\frac{2x}{x^2-4} = \infty$$

Moreover

$$lim_{x\to 2^-}x^2-4 = 0^-$$

and using the limit tables we get

$$lim_{x\to 2^-}\frac{2x}{x^2-4} = -\infty$$

Thus the left and right limits do not coincide implying that $lim_{x\to 2}\frac{2x}{x^2-4}$ does not exist.

22. $lim_{x\to 1}\frac{3x}{x^2+2x+1}$
How you should think in your draft/head: If I plug 1 we are all fine the denominator is not 0. So that is a super easy question.
How you can answer: The function defined by $\frac{3x}{x^2+2x+1}$ is continuous at 1 thus by definition of continuity we have

$$lim_{x\to 1}\frac{3x}{x^2+2x+1} = \frac{3\times 1}{1^2+2\times 1+1} = \frac{3}{4}$$

23. $lim_{x\to 1}\frac{x^2-25}{x^2-4x-5}$
How you should think in your draft/head: If I plug 1 we are all fine the denominator is not 0. So that is a super easy question.
How you can answer: The function defined by $\frac{x^2-25}{x^2-4x-5}$ is continuous at 1 thus by definition of continuity we have

$$lim_{x\to 1}\frac{x^2-25}{x^2-4x-5} = \frac{1^2-25}{1^2-4\times 1-5} = \frac{-24}{-8} = 3$$

24. $lim_{x\to 3}\frac{\sqrt{x^2-5}+2}{x-3}$
How you should think in your draft/head: If I plug 3 denominator and numerator are 0 oups indeterminate but we have a square root how about trying to rationalize and see.
How you can answer:

$$\begin{aligned}\frac{\sqrt{x^2-5}+2}{x-3} &= \frac{\sqrt{x^2-5}+2}{x-3}\frac{\sqrt{x^2-5}-2}{\sqrt{x^2-5}-2} = \frac{x^2-9}{(x-3)(\sqrt{x^2-5}-2)} \\ &= \frac{x(x-3)(x+3)}{(x-3)(\sqrt{x^2-5}-2)} = \frac{x(x+3)}{\sqrt{x^2-5}-2)}\end{aligned}$$

The function $\frac{x(x+3)}{\sqrt{x^2-5}-2)}$ is now continuous at 3 thus

$$lim_{x\to 3}\frac{\sqrt{x^2-5}+2}{x-3} = lim_{x\to 3}\frac{x(x+3)}{\sqrt{x^2-5}-2)} = \frac{3(3+3)}{\sqrt{3^2-5}-2)} = \frac{3\times 6}{2} = 9$$

25. $lim_{x\to 0}\frac{2^x+sin(x)}{x^4}$
How you should think in your draft/head: If I plug 0 the denominator is 0 but the numerator is 1. So we can see that this is not an indeterminate. Also, the denominator is always positive so I even do not have to separate left and right limits.
How you can answer: The function defined by $2^x + sin(x)$ is continuous at 0 thus

$$lim_{x\to 0}2^x + sin(x) = 2^0 + sin(0) = 1$$

and

$$lim_{x\to 0}x^4 = 0^+$$

Using the limit tables, we get

$$lim_{x\to 0}\frac{2^x+sin(x)}{x^4} = \infty$$

26. $lim_{x\to 1^-}\frac{1}{x-1} + e^{x^2}$
How you should think in your draft/head: That is a sum of two function and both are not hard to find a limit of.
How you can answer: $lim_{x\to 1^-}x-1 = 0^-$. Using the quotient limit table, we get

$$lim_{x\to 1^-}\frac{1}{x-1} = -\infty$$

and the function defined by e^{x^2} is continuous at 1 thus

$$lim_{x\to 1^-}e^{x^2} = e^{1^2} = e$$

Using the limit sum table,

$$lim_{x\to 1^-}\frac{1}{x-1} + e^{x^2} = -\infty$$

27. $lim_{x\to\infty}2x^2 - 3x$
How you should think in your draft/head: Limit of a polynomial super easy cool.
How you can answer:

$$2x^2 - 3x = x^2(2 - 3/x)$$

And

$$lim_{x\to\infty}2 - 3/x = 2 - 0 = 2$$

and

$$lim_{x\to\infty}x^2 = \infty$$

Using the product limit table, we get

$$lim_{x\to\infty}2x^3 - 3x = lim_{x\to\infty}x^2(2 - 3/x) = \infty$$

28. $lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2-x}}{x}$
How you should think in your draft/head: If I plug 0 in this expression I get 0 for denominator and numerator grrrr. Well I have a rational function, let try to rationalize, who knows.
How you can answer:

$$\begin{aligned}\frac{\sqrt{x+2}-\sqrt{2-x}}{x} &= \frac{\sqrt{x+2}-\sqrt{2-x}}{x}\times\frac{\sqrt{x+2}+\sqrt{2-x}}{\sqrt{x+2}+\sqrt{2-x}}\\ &= \frac{2x}{x(\sqrt{x+2}+\sqrt{2-x})}\\ &= \frac{2}{(\sqrt{x+2}+\sqrt{2-x})}\end{aligned}$$

The function defined by $\frac{2}{(\sqrt{x+2}+\sqrt{2-x})}$ is now continuous at 0 thus

$$lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2-x}}{x} = lim_{x\to 0}\frac{2}{(\sqrt{x+2}+\sqrt{2-x})} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

29. $lim_{x\to 0^+}\frac{e^x}{1+ln(x)}$
How you should think in your draft/head: If I try quickly to work out the limit here the numerator goes to 1 and the denominator to $-\infty$ so no problem here it will be a walk in a park.
How you can answer: We have that e^x is continuous at 0 thus

$$lim_{x\to 0^+}e^x = e^0 = 1$$

and it is known that

$$lim_{x\to 0^+}ln(x) = -\infty$$

thus

$$lim_{x\to 0^+}1+ln(x) = -\infty$$

And using the quotient limit table we get

$$lim_{x\to 0^+}\frac{e^x}{1+ln(x)} = 0$$

30. $lim_{x\to\infty}\sqrt{x^2+1}-2x$
How you should think in your draft/head: If I try to do the naive way I get infinity summed with - infinity, not great it is an indeterminate. There is a square root I cold rationalize but I could also try to first factorize by higher power of x and see what happens, maybe I do not need to rationalize. Indeed, if you try quickly you will see that factorizing the higher power already solve the problem so if you rationalized you lost good time.
How you can answer: Since we are look at the limit at infinity we can look at the function close to ∞ for instance in $(0,\infty)$ it is enough. We have for any $x>0$,

$$\sqrt{x^2+1}-2x = \sqrt{x^2(1+1/x^2)}-2x = \sqrt{x^2}\sqrt{(1+1/x^2)}-2x = |x|\sqrt{(1+1/x^2)}-2x = x(\sqrt{(1+1/x^2)}-2)$$

since $x>0$.
We have

$$lim_{x\to\infty}\sqrt{(1+1/x^2)}-2 = \sqrt{1+0}-2 = -1 < 0$$

and

$$lim_{x\to\infty}x = \infty$$

We can use the product limit table and get

$$lim_{x\to\infty}\sqrt{x^2+1}-2x = lim_{x\to\infty}x(\sqrt{(1+1/x^2)}-2) = -\infty$$

31. $lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$
How you should think in your draft/head: When I plug 1 on denominator and numerator I get 0 for both, boh it it is again an indeterminate. I need to factor somehow. You can see that $\sqrt{x} = \sqrt[3]{x}^3$, also

$$x^3-1 = (x-1)(x^2+x+1)$$

. So maybe I can do it.
How you can answer:

$$\sqrt{x}-1=(\sqrt[3]{x}-1)(\sqrt[3]{x}^2+\sqrt[3]{x}+1)$$

Thus

$$\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\frac{\sqrt[3]{x}-1}{(\sqrt[3]{x}-1)(\sqrt[3]{x}^2+\sqrt[3]{x}+1)}=\frac{1}{\sqrt[3]{x}^2+\sqrt[3]{x}+1}$$

The function defined by $\frac{1}{\sqrt[3]{x}^2+\sqrt[3]{x}+1}$ is continuous at 1 thus

$$lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=lim_{x\to 1}\frac{1}{\sqrt[3]{x}^2+\sqrt[3]{x}+1}=\frac{1}{\sqrt[3]{1}^2+\sqrt[3]{1}+1}=\frac{1}{3}$$

4. Find the following limits involving absolute values

(a) $lim_{x\to 1}\frac{x^2-1}{|x-1|}$

How you should think in your draft/head: I have an absolute value how can I do. Well lets try first to do a simple replacement and see what happens. Well the denominator and numerator are 0 so indeterminate I need to simplify. Well with the absolute value of $|x-1|$ I see that this expression depends on $x>1$ or $x<1$ so I will need to separate the limit on the right and left if I want to get somewhere.

How you can answer: When $x>1$

$$\frac{x^2-1}{|x-1|}=\frac{(x-1)(x+1)}{x-1}=x+1$$

Thus

$$lim_{x\to 1^+}\frac{x^2-1}{|x-1|}=lim_{x\to 1^+}x+1=2$$

When $x<1$,

$$\frac{x^2-1}{|x-1|}=\frac{(x-1)(x+1)}{-(x-1)}=-(x+1)$$

Thus

$$lim_{x\to 1^-}\frac{x^2-1}{|x-1|}=lim_{x\to 1^-}-(x+1)=-2$$

We see that the limit on the right an left do not coincide thus $lim_{x\to 1}\frac{x^2-1}{|x-1|}$ does not exist.

(b) $lim_{x\to -2}\frac{1}{|x+2|}+x^2$

How you should think in your draft/head: Let first see as usual what happens when I plug -2, well x^2 no problem for $\frac{1}{|x+2|}$ the numerator 1 and the denominator is 0 but positive since the absolute value is always positive cool I know this limit with the quotient limit table.

How you can answer:

$$lim_{x\to -2}x^2=(-2)^2=4$$

and

$$lim_{x\to -2}|x+2|=0^+$$

thus using the quotient limit table we get

$$lim_{x\to -2}\frac{1}{|x+2|}=\infty$$

and using the sum limit table we get

$$lim_{x\to -2}\frac{1}{|x+2|}+x^2=\infty$$

(c) $lim_{x\to 3^-}\frac{x^2|x-3|}{x-3}$

How you should think in your draft/head: Let first see as usual what happens when I plug 3, well I get 0 in numerator and denominator, indeterminate grrrr. Well it is not so bad as when you close to 3 on the left (meaning $x<3$), $|x-3|=-(x-3)$, I should be able to simplify then.

How you can answer: When $x<3$,

$$\frac{x^2|x-3|}{x-3}=\frac{-x^2(x-3)}{x-3}=-x^2$$

thus

$$lim_{x\to 3^-}\frac{x^2|x-3|}{x-3}=lim_{x\to 3^-}-x^2=-3^2=-9$$

5. Find the value of the parameter k to make the following limit exist and be finite. What is then the value of the limit?

$$lim_{x\to 5}\frac{x^2+kx-20}{x-5}$$

6. Answer the following questions for the piecewise defined function f described bellow.

$$\begin{array}{rccl} f: & \mathbb{R}\backslash\{1\} & \to & \mathbb{R} \\ & x & \mapsto & \left\{\begin{array}{cl} sin(\pi x) & for\ x<1 \\ 2^{x^2} & for\ x>1 \end{array}\right. \end{array}$$

(a) $f(1)=$

Solution: not defined as it is

(b) $lim_{x\to 0}f(x)=$

Solution: Since $0<1$ and the function defined by $sin(\pi x)$ continuous at 0

$$lim_{x\to 0}f(x)=lim_{x\to 0}sin(\pi x)=sin(\pi 0)=0$$

(c) $lim_{x\to 1}f(x)=$

How you should think in your draft/head: Here you have a piecewise function that changes around 1 thus we will need to separate the limit on the right and left and compare them.

How you can answer:

$$lim_{x\to 1^+}f(x)=lim_{x\to 1^+}2^{x^2}=2^{1^2}=2$$

since the function defined by 2^{x^2} is continuous at 1.

$$lim_{x\to 1^-}f(x)=lim_{x\to 1^+}sin(\pi x)=sin(\pi)=0$$

since the function defined by $sin(\pi x)$ is continuous at 1.

Since the limit on right and left do not coincide then $lim_{x\to 1}f(x)$ does not exist.

7. Answer the following questions for the piecewise defined function f described bellow.

$$\begin{array}{rccl} f: & (-\infty,-2)\cup(-1,0)\cup(0,1)\cup(1,\infty) & \to & \mathbb{R} \\ & t & \mapsto & \left\{\begin{array}{cc} t^2 & for\ t<-2 \\ \frac{t+6}{t^2-t} & for\ -1<t<2 \\ 3t-2 & for\ t\geq 2. \end{array}\right. \end{array}$$

(a) $f(-3/2)=$

Solution $-1<-3/2<2$, thus

$$f(-3/2)=\frac{-3/2+6}{(-3/2)^2-(-3/2)}=\frac{4\times 9}{2\times 15}=\frac{6}{5}$$

(b) $f(2)=$

Solution

$$f(2)=3\times 2-2=4$$

(c) $f(3/2) =$

Solution $-1 < 3/2 < 2$, thus

$$f(3/2) = \frac{3/2+6}{(3/2)^2-(3/2)} = \frac{4\times 15}{2\times 3} = 10$$

(d) $lim_{t\to -2} f(t) =$

Solution As the function is defined this limit only exist on the left so this limit does not exist at -2.

(e) $lim_{t\to 1^+} f(t) =$

When we are close to 1 on the right the function is equal to $\frac{t+6}{t^2-t}$. Moreover when $t > 1$, $t(t-1) > 0$. Thus

$$lim_{t\to 1^+} t^2 - t = lim_{t\to 1^+} t(t-1) = 0^+$$

and

$$lim_{t\to 1^+} t + 6 = 7 > 0$$

Thus, we can apply the quotient limit table and we get

$$lim_{t\to 1^+} f(t) = lim_{t\to 1^+} \frac{t+6}{t^2-t} = \infty$$

(f) $lim_{t\to 2} f(t) =$

Solution:

How you should think in your draft/head: Here the function around 2 has different values depending if you are on the left or right. We will have to study separately the left and right limit and compare then to get an answer.

How you can answer:

The function defined by $\frac{t+6}{t^2-t}$ is continuous at 2 thus

$$lim_{t\to 2^-} f(t) = lim_{t\to 2^-} \frac{t+6}{t^2-t} = \frac{2+6}{2^2-2} = 4$$

Moreover,

$$lim_{t\to 2^+} f(t) = lim_{t\to 2^+} 3t - 2 = 4$$

Thus the limit on the right and left coincide and

$$lim_{t\to 2} f(t) = lim_{t\to 2^+} f(t) = lim_{t\to 2^-} f(t) = 4$$

(g) $lim_{t\to 0} f(t) =$

Solution

$-1 < 0 < 2$ thus

$$lim_{t\to 0} f(t) = lim_{t\to 0} \frac{t+6}{t^2-t}$$

$$lim_{t\to 0} t + 6 = 6 > 0$$

and when $0 < t < 1$,

$$t^2 - t = t(t-1) < 0$$

thus

$$lim_{t\to 0^+} t^2 - t = 0^-$$

Hence, using the quotient limit table we get

$$lim_{t\to 0^+} \frac{t+6}{t^2-t} = -\infty$$

When $-1 < t < 0$,

$$t^2 - t = t(t-1) > 0$$

thus

$$lim_{t\to 0^-} t^2 - t = 0^+$$

Hence, using the quotient limit table we get

$$lim_{t\to 0^-} \frac{t+6}{t^2 - t} = \infty$$

Thus the limits on the right and left at 0 do not coincide and $lim_{t\to 0} f(t)$ does not exist.